Perpetual leapfrogging in international competition

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Abstract

Technological leadership has shifted at various times from one country to another. We propose a mechanism that explains this perpetual cycle of technological leapfrogging by incorporating knowledge spillovers into a two-country model of innovation including the dynamic optimization of an infinitely lived consumer. In the model, the stock of knowledge accumulates in each country over time because of domestic innovation and spillovers from foreign innovation, while spillovers take place through imitation and foreign direct investment. We show that if the rate of imitation is high, only the technologically leading country innovates in equilibrium (a North–South regime) where leapfrogging never arises. Conversely, if the imitation rate is sufficiently low, both countries innovate in equilibrium (a North–North regime), and so technological leadership may shift first from one country to another, and then if the international spillovers are sufficiently efficient, may perpetually alternate between the two along an equilibrium path.

JEL Classification Numbers: E32, F44, O33

Keywords: Perpetual leapfrogging; endogenous selection of North–South and North–North models, endogenous innovation; spillovers

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1 Introduction

Throughout history, technological leadership has shifted at various times from one country to another. For instance, during the early 17th century, Venice and Spanish Lombardy were among the technologically most advanced regions in Europe (Davids 2008, p. 2). Over the centuries, the "technological center of gravity of Europe then moved, residing at various times in Italy, southern Germany, the Netherlands, France, England, and then again in Germany" (Mokyr 1990, p. 207). Some economic historians even claim that the US had begun to lose its technological leadership as early as the early 1990s (Nelson and Wright 1992).

An important question is why such economic and technological leapfrogging takes place. An equally fundamental question is why technological leapfrogging has repeatedly occurred. To respond to the first question, Brezis, Krugman, and Tsiddon (1993) provided an economic explanation based on major exogenous changes in technology. When such change occurs, the new technology appears less productive for leading nations, given their extensive experience with older technologies. Lagging nations with less experience will then introduce the new technology. As these accumulate sufficient experience with the new technology, the leapfrogging of technological leadership occurs. We may apply this same theory to the second question by considering the perpetual cycles of leapfrogging as responses to the perpetual changes in technology. This explanation is, however, essentially exogenous, as it is based on macro shocks in technology. Although a variety of studies have followed Brezis, Krugman, and Tsiddon, no existing work formally provides a fully endogenous explanation that responds to both of these questions.

The aim of this analysis is to develop a fully endogenous theory that explains both the leapfrogging of technological leadership and the perpetual cycle in technological leadership as a market-driven equilibrium phenomenon. For this purpose, we focus on international knowledge spillovers in a two-country dynamic general equilibrium model of endogenous innovation with the dynamic optimization of consumption and saving by an infinitely lived consumer. As the firms in a country develop innovations, the stock of knowledge accumulates in the home country, and this subsequently but only partially contributes to the accumulation of foreign knowledge because of international spillovers through imitation and foreign direct investment.[1]

By regarding technological leadership as the state whereby a given country innovates most among all countries, we demonstrate that technological leadership by that country may shift to another country and then may perpetually alternate between the countries. Specifically, we obtain two main results. (a) If the imitation rate in a technologically lagging country is sufficiently high, only the leading country innovates in equilibrium as in a North–South product-cycle model à la Krugman (1979) and Helpman (1993) (a North–South regime), in which case leapfrogging never takes place. (b) If the imitation rate is sufficiently low, the world behaves like a North–North model, in which both leading and lagging countries engage in innovation (a North–North regime). In this case, technological leadership may shift over time and will perpetually move back and forth between countries along an equilibrium path if the international knowledge spillovers are

[1] This approach follows a number of related theoretical models (Brezis and Tsiddon 1998; van de Klundert and Smulders 2001; Desmet 2002). As argued by Brezis (1995), foreign capital plays a role in industrialization and development processes. We may also accept that international capital flows, as well as imports, are an important channel for international knowledge spillover, as discussed in the literature (Grossman and Helpman 1991; Feenstra 1996). See Branstetter (2006) for recent empirical evidence.

sufficiently efficient.

The key driving force behind perpetual leapfrogging is the ability of a country to learn from abroad. For example, a lagging country may learn much more from foreign innovations developed in the leading country than the leading country learns from those in the lagging country. Meanwhile, domestic innovations take place that accumulate the knowledge stock in each country. The analysis formally shows that leapfrogging is possible only in the North–North regime, where the lagging country has a dual engine of knowledge growth consisting of domestic innovation and foreign innovation diffusing through imitation and foreign direct investment. If a country can learn efficiently from diffuse foreign innovations, technological leadership will perpetually alternate between the countries.

The important mechanism that we identify in the paper is that the North–North regime is possible only when imitation in the lagging country is less active. This is because the profitability of a new innovation is sufficiently large that innovation pays, even for the technologically lagging country when the imitation rate is sufficiently low. Where imitation is active, the lagging country does not innovate and simply receives the spillovers from foreign innovation through imitation and foreign direct investment. In this North–South regime, leapfrogging never arises as the spillovers by themselves can only make the lagging South, at most, as innovative as the leading North but *not more* innovative.

The striking implication of this result is that active imitation in the lagging country hampers its ability to leapfrog the leading country, even if imitation is a channel for knowledge spillovers. We then say that imitation is only useful for the lagging country to catch up with the leading country. Unless the lagging country shifts to an innovative economy in a North–North regime by discouraging imitation, leapfrogging can never occur in equilibrium.

In order to capture these cyclical phenomena in the simplest fashion, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999), and Matsuyama (1999, 2001) by assuming that patents last only for a single period in a discrete time model. This assumption implies that a unit period is sufficiently long, which can be somewhere around 20 years. Given that in reality, many innovated consumption goods become obsolete before their patents expire, for the sake of simplicity, we assume that innovations are made obsolete in a single period (which is fairly long). In line with existing studies, we assume the temporary nature of the monopoly enjoyed by innovators, which plays a role in explaining perpetual leapfrogging.[2]

Our analysis relates most to studies in international economics following the work in Brezis, Krugman, and Tsiddon (1993). Most closely related is a paper by van de Klundert and Smulders (2001), which focuses on the international capital market using an endogenous growth model. By allowing for nontradable goods, capital flows, and endogenous innovations, van de Klundert and Smulders (2001) explain the well-documented observation that a leading country (e.g., England) tends to lose its technological leadership by becoming a rentier economy that invests in a new technologically leading country. However, as in the other related studies described below, their analysis does not address the second question of why the leapfrogging of technology leadership produces a perpetual cycle. In an earlier contribution, Brezis and Tsiddon (1998) show that capital mobility

[2]See also Iwaisako and Tanaka (2012) for endogenous cycles in a North–South product-cycle model with overlapping generations, in which innovation and imitation interact with each other to generate perpetual fluctuations in the world growth rate. In their model, however, there is no leapfrogging.

may spur leapfrogging. Desmet (2002) extends the Ricardian model in Brezis, Krugman, and Tsiddon (1993) to a Heckscher–Ohlin framework by introducing mobile capital and spillovers. Desmet then specifies a mechanism by which the most advanced country may reinforce its dominant position by adopting the new technology if spillovers between the old and new technologies are sufficiently strong, which weakens the opportunity for lagging nations to take off and leapfrog.

In a different context, the literature on industrial organization has clarified the conditions for leapfrogging. For example, Motta, Thisse, and Cabrales (1997) illustrate in a model with vertical product differentiation that free trade may either encourage or reverse quality leadership. The present study extends these existing analyses by explicitly and formally providing a fully endogenous explanation of why leapfrogging takes place perpetually using a single factor, namely, international knowledge spillovers. Ohyama and Jones (1995) provide a similar explanation for leapfrogging by firms, with a focus on comparative advantage. They argue that lagging regions typically have a comparative advantage in the new technology as the leading country has greater experience in the older technologies. This provides lagging regions with an opportunity to adopt the new technology first.

The endogenous occurrence of perpetual leapfrogging is not new in the context of price competition between firms. For instance, Giovannetti (2001) considers a duopoly in which firms considering infinite technological adoption set prices with Bertrand competition in the product market. Using this model, Giovannetti identifies the conditions whereby firms alternate in adopting the new technology, thereby representing a leapfrogging process. He shows that demand conditions, such as price elasticity, play a role in determining whether leapfrogging can be perpetual in Bertrand competition.[3] Lee, Kim, and Lim (2011) have provided recent empirical support for this contention.

In demonstrating the cyclical occurrence of leapfrogging, we reveal that the dynamic general equilibrium of the model is characterized by a simple nonlinear dynamic system in discrete time. Along an equilibrium path generated by this system, we show that technological leadership may fluctuate perpetually. Nonlinear equilibrium dynamics in discrete time provide a useful tool for describing complicated, real-world economic phenomena (Nishimura and Yano 2008), which may include the perpetual cycle of leapfrogging. Our analysis extends this line of research by demonstrating the possibility of a perpetual cycle of leapfrogging using a one-dimensional nonlinear difference equation.

2 The model

Time is discrete and extends from $-\infty$ to $+\infty$. Consider two countries, A and B, which have identical preferences and differ only in their initial levels of innovation productivity. The countries are denoted by i or k ($i = A,\ B$; $k = A,\ B$), using a superscript for variables pertaining to the production side and a subscript for those pertaining to the consumption side.

There is a continuum of differentiated consumption goods in each period t. Each good is indexed by j. Given that we later allow for foreign direct investment (FDI), the country where a particular firm innovates and manufactures may change. Let $\Gamma^i(t)$ be the set of goods that are innovated in country i in period t, and let $\Lambda^i(t)$ be the set of goods that are manufactured in country i in period t.

[3] See Aghion et al. (2002) for perpetual leapfrogging at the firm level.

2.1 Consumption

In each country, an infinitely lived representative consumer inelastically supplies L units of labor for production and R&D in every period. Note that the two countries are assumed to have equal labor forces, L. Each consumer is endowed with the same intertemporal utility function

$$U_i = \sum_{t=0}^{\infty} \beta^t \ln u_i(t),$$

where $\beta \in (0,1)$ is the time preference rate. The temporary utility $u_i(t)$ is defined on the set $\{\Lambda^A(t) \cup \Lambda^B(t)\}$ of goods manufactured in both countries (free trade), taking the standard Dixit–Stiglitz form:

$$u_i(t) = \left(\int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} x_i(j,t)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{1}$$

where $x_i(j,t)$ is the consumption of good j in country i. Parameter $\theta \in (0,1)$ denotes an inverse measure of the elasticity of substitution. Let $E_i(t) \equiv \int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} p(j,t) x_i(j,t) dj$ be the spending in country i, where $p(j,t)$ denotes the price of good j. Solving the utility maximization problem in (1) leads to the demand function for good j as $x_i(j,t) = p(j,t)^{-(1/\theta)} E_i(t)/P(t)^{1-(1/\theta)}$, where $P(t)$ is the price index.[4] Aggregating these expressions, we obtain the derived aggregate demand, $x_A(j,t) + x_B(j,t) \equiv x(j,t)$, as

$$x(j,t) = \frac{E(t) p(j,t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}}, \tag{2}$$

where $E(t) = E_A(t) + E_B(t)$ is the aggregate spending in period t. The price elasticity of demand is constant at θ^{-1} for any j.

Solving the dynamic optimization of the consumer's utility for consumption and saving decisions under the intertemporal budget constraint results in the usual Euler equation $E_i(t+1)/E_i(t) = \beta(1 + r(t))$, where $r(t)$ is the interest rate in period t. We then obtain

$$\frac{E(t+1)}{E(t)} = \beta(1 + r(t)). \tag{3}$$

2.2 Innovation, imitation, FDI, and manufacturing

A single firm innovates and monopolistically supplies each differentiated consumption good, in line with the R&D-based growth model with expanding variety (Romer 1990).[5] Innovating a new good takes one period. When an R&D firm in country i invests $1/A^i(t-1)$ units of labor in period $t-1$, it innovates a new good *at the end of* period $t-1$.[6] Here $A^i(t-1)$ denotes the technology level in innovation for country i in period $t-1$.

[4] As is well known, the index is defined as $P(t) = \left(\int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} p(j,t)^{1-(1/\theta)} dj \right)^{\frac{1}{1-(1/\theta)}}$.

[5] To make the analysis as simple as possible, consistent with the standard endogenous growth model, we basically assume that innovation and manufacturing are by the same firm. Hence, there is no adoption of innovation or an explicit market for innovation. However, without any change in the results, it is possible to assume a more general model with an innovation market in which innovation and manufacturing are by different firms.

[6] We can extend the current deterministic innovation process to a stochastic form.

Innovations may diffuse internationally through two channels. The first channel is exogenous imitation in a technologically lagging country, as in Helpman (1993). Here innovations developed at the end of period $t-1$ are imitated in the subsequent period t at a probability of ι. Imitated innovations are then manufactured by monopolistic firms in the lagging country where imitation takes place. For innovations that are not imitated, the original innovating firm sets up a production plant. The firm will then choose the country in which to manufacture the good in order to maximize profits. In equilibrium, as foreign profits may be greater, the firm may transfer production to a foreign country through FDI. This is the second channel for innovation diffusion.[7]

Each monopolistic firm produces $x(j,t)$ units of good j using domestic labor as the input. Assume that there are constant returns to scale in the production of good j and that the productivity of labor is the same in both countries, which is normalized to be one.[8] The marginal cost is thus equal to the wage rate in country i, $w^i(t)$.

When the firm that innovates technology for good j at the end of period $t-1$ chooses to manufacture in country i, captured by $j \in \Lambda^i(t)$, it maximizes monopolistic profit in period t by setting a price at $p(j,t) = w^i(t)/(1-\theta) \equiv p^i(t)$, taking into account the constant price elasticity $1/\theta$ according to (2). This monopolistic price does not depend on the country for innovation, only on the country for manufacture. With this monopolistic price $p^i(t)$, we can derive from (2) the demand and profit functions as

$$x(j,t) = \frac{E(t)p^i(t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}} \equiv x^i(t) \tag{4}$$

and

$$\pi(j,t) = \theta E(t) \left(\frac{p^i(t)}{P(t)} \right)^{1-(1/\theta)} \equiv \pi^i(t) \tag{5}$$

for $j \in \Lambda^i(t)$ $(i = A,\ B)$. As firms prefer the country where profits are higher, the discounted expectation of a value of the firm innovating in country i is expressed as

$$V^i(t-1) = (1-\iota)\,\frac{\max\{\pi^A(t), \pi^B(t)\}}{1+r(t-1)} - \frac{w^i(t-1)}{A^i(t-1)}, \tag{6}$$

in which $(1-\iota)$ denotes the probability for an innovating firm to escape imitation.

In order to capture cyclical phenomena in the simplest fashion possible, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999), and Matsuyama (1999, 2001) by assuming that patents last only for one period. This assumption implies that the length of a unit period is sufficiently long, which can be somewhere around 20 years given the duration of real-world patents. Given that in reality, many

[7] In line with the literature on international trade and growth (Lai 1998), we do not distinguish between the various forms of production transfer, including fully and partly owned subsidiaries and licensing. For simplicity, we assume that imitation occurs before choosing the country in which to manufacture, so that all innovations are under threat of imitation. However, even if we considered that only the innovations manufactured in the lagging country may be imitated, our results shown below would not change qualitatively.

[8] Here we simply consider that efficiency in manufacturing normalizes across countries. We can extend this simple setting by allowing for country-specific manufacturing efficiency and endogenous technological progress. In such an extended model, we can easily verify that the comparative advantage between R&D and manufacturing (rather than the absolute advantage in R&D) plays an important role in perpetual leapfrogging, although the results and their implications for perpetual leapfrogging do not fundamentally change.

innovated consumption goods become obsolete before their patent expires, we may assume that innovations are made obsolete within a single period (which in our model is fairly long).[9] As shown below, this assumption makes the analysis tractable without any fundamental change in the results. Finally, free entry guarantees that the net value of a firm should not be positive in equilibrium: $V^i(t-1) \leq 0$ for each i.

2.3 Knowledge accumulation and spillovers

Technology in innovation $A^i(t)$ advances with knowledge accumulation. Following Romer (1990), we assume intertemporal knowledge spillovers in innovation: current innovations contribute to accumulation of the stock $A^i(t)$ of knowledge, with which the cost of innovation, $1/A^i(t)$, reduces over time. Here, as is standard, we consider that the technology level in innovation $A^i(t)$ is interpreted as the knowledge stock in innovation.

The knowledge stock for a country consists of cumulative innovations of two types: home and foreign innovations. Denote by $N^i(t) \equiv \int_{\Gamma^i(t)} dj$ the number of innovations developed in country i in period t. As in Romer (1990), we assume that the knowledge stock $A^i(t)$ linearly depends on a sum of domestic innovations that are developed up to the beginning of period t; i.e., $N^i(t-1) + N^i(t-2) + \cdots$, where $N^i(s)$ is a proxy for the flow of knowledge generated as a by-product of the innovations in period s. Then, the international knowledge spillovers accompany FDI or imitation, such that we assume that each country learns from its foreign innovation inflows. The knowledge stock for country i also depends on a sum, $M^i(t-1)+M^i(t-1)+\cdots$, in which $M^i(s) \equiv \int_{j\in\Gamma^k(s-1)\cap\Lambda^i(s)} dj$ is the amount of foreign innovation that is made in period $s-1$ and then flows into country i from country k in period s. Accordingly, we simply describe the knowledge stock using

$$A^i(t) = \sum_{s=-\infty}^{t} \left(N^i(s-1) + \mu M^i(s-1)\right), \tag{7}$$

where $\mu \in [0,1]$ captures the efficiency of the contribution of international knowledge spillovers through foreign innovation inflows to knowledge accumulation and whereby technological progress occurs. The efficiency of international knowledge spillovers increases with μ. If $\mu = 1$, spillovers are as efficient as domestic spillovers; if $\mu = 0$, there is no learning at all from foreign innovations.

3 Perpetual leapfrogging

In this section, we prove the main result that technological leadership may fluctuate over time, thereby perpetually moving back and forth between countries. Before proceeding, we provide a formal definition of the concept of technological leadership. Taking into account the notion in economic history (Davids 2008),[10] we refer to a country that develops the most innovations among the countries as the technological leader, and a country that develops few innovations as a lagging country. In the present model, and as will be

[9] This assumption may also be justified if each innovation is interpreted as fairly specific. For example, "innovation" in this model would be represented by the specific innovation associated with iPhone 4S or smart phones instead of cell phones or information technology more generally.

[10] Davids (2008) considered that a country that has technological leadership plays an initiating role in the development of new technologies across a wide variety of fields.

made apparent later, this definition is equivalent to that in trade theory (Brezis, Krugman, and Tsiddon 1993), which defines leadership as the state whereby a given country has the highest productivity among the countries. Thus, in equilibrium, country i innovates more if and only if its innovation productivity is higher; $N^i(t) > N^k(t)$ if and only if $A^i(t) > A^k(t)$. For simplicity, we use $A^i(t) > A^k(t)$ to designate country i as the technological leader, and we refer to any reversal of the leading position as technological leapfrogging.

Without loss of generality, we suppose that country A is the leading country in period t, $A^A(t) > A^B(t)$ (and thus $N^A(t) > N^B(t)$ to be shown in equilibrium), and we refer to this situation as regime A. If $A^A(t) < A^B(t)$ (and thus $N^A(t) < N^B(t)$ to be shown in equilibrium), we refer to it as regime B.

In any period of time, this model can be regarded as a variant of a conventional two-good Ricardian model, where the two outputs considered are innovation and production. Given $A^A(t) > A^B(t)$, there are potentially three possible specialization patterns in period t: (1) one in which both countries engage in manufacturing, (2) one in which both countries engage in R&D, and (3) one in which both countries are specialized. It is useful to define a new variable, $N(t) = \int_{j\in\{\Lambda^A(t)\cup\Lambda^B(t)\}} dj$, which is the total number of goods manufactured in t, satisfying $N(t) = N^A(t-1) + N^B(t-1)$.

3.1 North–South regime

Suppose that both countries produce goods. In this case, only the leading country A innovates, and both countries manufacture. Then we have $N^A(t) > 0$ and $N^B(t) = 0$; $N(t+1) = N^A(t)$. As this situation is similar to the North–South product-cycle model à la Krugman (1979) and Helpman (1993) where only the North innovates and both the North and the South manufacture, we may refer to this pattern as a "North–South regime," in which the leading country corresponds to the North and the lagging country to the South. As manufacturing takes place in both countries, the wages are internationally equated, $w^A(t) = w^B(t) = w(t)$, implying $p^i(t) = p(t)$ and thus $x^i(t) = x(t)$ by (4).

Only the leading country innovates, so the free-entry condition holds as $V^A(t) = 0 > V^B(t)$. By (5) and (6), with the Euler equation (3), $V^A(t) = 0$ implying

$$\beta\theta(1-\iota)\frac{E(t)}{N^A(t)} = \frac{w(t)}{A^A(t)}, \tag{8}$$

where use has been made of $N^A(t) = N(t+1)$. Equation (8) ensures that the discounted expectation value of an innovation (left-hand side) and the cost (right-hand side) are balanced. The condition of $0 > V^B(t)$ implies $A^A(t) > A^B(t)$. This is an important condition ensuring that country A is the leading country in the North–South regime.

We can describe the labor market-clearing conditions as

$$L = \frac{N^A(t)}{A^A(t)} + \alpha^A(t)N(t)\ x(t) \tag{9}$$

for the leading country A and

$$L = \alpha^B(t)N(t)\ x(t) \tag{10}$$

for the lagging country B, in which $\alpha^i(t)$ denotes an endogenous fraction of goods manufactured in country i in all goods $N(t)$, with $\sum_{i=A,B}\alpha^i(t) = 1$. Because $x^i(t) = x(t) \equiv$

$(1-\theta)E(t)/\left(w(t)N(t)\right)$ by (4), the labor conditions (9) and (10) can be combined into a single world labor constraint as

$$2L = \frac{N^A(t)}{A^A(t)} + (1-\theta)\frac{E(t)}{w(t)}. \tag{11}$$

The left-hand side is the world supply of labor, and the right-hand side is the world demand for labor from both the innovation sector in leading country A and the manufacturing sectors in both countries.

We can eliminate the term $E(t)/w(t)$ from the world labor market-clearing condition (11), using the free-entry condition (8). Then, the flow of innovation in period t is derived as

$$N^A(t) = A^A(t)\frac{2L\Theta\,(1-\iota)}{1+\Theta\,(1-\iota)}, \tag{12}$$

where $\Theta \equiv \beta\theta/(1-\theta)$. Equation (12) shows that the innovation flow $N^A(t)$ increases with the knowledge stock $A^A(t)$ and the time preference β, and decreases with the elasticity of substitution θ^{-1}. By (8) and (10), with (12), the fraction of goods that are manufactured in each country is obtained as

$$\alpha^A(t) = \frac{1-\Theta\,(1-\iota)}{2} \text{ and } \alpha^B(t) = \frac{1+\Theta\,(1-\iota)}{2}. \tag{13}$$

To ensure that the leading country A manufactures, i.e., $\alpha^A(t) > 0$, we need to impose

$$\Theta\,(1-\iota) < 1.^{11} \tag{14}$$

This requires that the imitation rate ι is high, the time preference β is small, and the elasticity of substitution θ^{-1} is high.

So far, we have four important conditions: two inequalities and two equations characterizing the North–South regime. Inequality (14) guarantees that the world falls into the North–South regime. Inequality $A^A(t) > A^B(t)$ requires that, in the North–South regime, country A becomes the leading country. Equations (12) and (13) determine the innovation flow and the fractions of manufactured goods, respectively.

It is important to discern the two roles for imitation in the North–South regime. First, as (12) shows, the innovation flow $N^A(t)$ decreases with the imitation rate ι. This reflects the usual negative effect of imitation, which reduces the expected value of innovation.[12] Second, as (13) shows, the manufacturing fraction $\alpha^A(t)$ in the leading country A increases with the imitation rate ι. This may seem odd at first, but it is due to the coexistence of FDI and imitation, as in Lai (1998). More specifically, imitation in the lagging country B discourages not only innovation but also FDI into the lagging country B.[13] Then, the

[11] This condition also guarantees that the resource condition for the leading country A, $N^A(t)/A^A(t) < L$, is satisfied.

[12] This negative relationship between imitation and innovation is intuitive but differs from that in Helpman (1993) where an increase in the imitation rate in the South increases the innovation rate in the North. This difference basically arises from the presence of endogenous FDI in our model. In Helpman's model, imitation shifts manufacturing from the North to the South, which opens up more resources for innovation in the North. In our model, as imitation increases manufacturing in the North, $\alpha^A(t)$, through the mechanism described later, the relationship can be negative, in line with Lai's (1998) result.

[13] The following specific mechanism works in our model. Imitation decreases the expected profitability and thus the demand for innovation. The labor demand from innovation declines, so that the wage rate in the leading country tends to decrease, discouraging the incentive for firms in the leading country to undertake FDI in the lagging country.

decreased innovation and FDI result in a shift of resources in the leading country A from innovation to manufacturing, resulting in an increase in $\alpha^A(t)$.

In what follows, we demonstrate that in the present North–South regime, leapfrogging never takes place. even if the spillovers are completely efficient ($\mu = 1$). To do so, from (7), we can easily write the flow of the knowledge stock as

$$A^i(t+1) - A^i(t) = N^i(t) + \mu M^i(t). \tag{15}$$

By (12) and (15), the growth of knowledge can be expressed as follows:

$$A^A(t+1) = \left(\frac{2L\Theta\,(1-\iota)}{1+\Theta\,(1-\iota)} + 1 \right) A^A(t) \tag{16}$$

and

$$A^B(t+1) = \mu M^B(t) + A^B(t). \tag{17}$$

As $A^A(t)$ is given by history, (16) fully determines the growth of knowledge for the leading country A. Apparently, (17) does not determine $A^B(t+1)$ without any additional historical assumption because the level of $M^B(t) = \alpha^B(t)N(t)$ depends on $N(t) = N^A(t-1) + N^B(t-1)$, which is determined by innovation activities made in the previous period, $t-1$. So far, we do not have any assumption on innovation activities in period $t-1$ or before. Nevertheless, as shown in our first theorem, regardless of innovation activities in the past, leapfrogging never takes place in the North–South regime.[14]

Theorem 1 (No Leapfrogging in the North–South Regime) *Suppose* $\Theta\,(1-\iota) < 1$. *Then, under the infinitely lived agent's dynamic optimization, the world is in the North–South regime, where only the leading country innovates. In this case, leapfrogging never takes place in equilibrium, and the roles of the countries as the leading North and the lagging South never reverse.*

Proof. By (14), $\Theta(1-\iota) < 1$ ensures the North–South regime. By (13), $M^B(t) = (1+\Theta\,(1-\iota))\,N(t)/2$. (a) Assume $A^A(t-1) > A^B(t-1)$. By the expression of $N^A(t-1)$ in (12), with $A^A(t-1) = N^A(t-1) + A^A(t)$ from (15), substituting $N(t) = N^A(t-1)$ into (17) derives

$$A^B(t+1) = A^A(t)\frac{\mu L\Theta\,(1-\iota)\,(1+\Theta\,(1-\iota))}{(2L+1)\,\Theta\,(1-\iota)+1} + A^B(t). \tag{18}$$

From (16) and (18), we can show that $A^A(t+1) > A^B(t+1)$ holds so long as $A^A(t) > A^B(t)$, noting $\Theta\,(1-\iota) < 1$ and $\mu < 1$. (b) Assume $A^A(t-1) < A^B(t-1)$. By symmetry, noting $N(t) = N^B(t-1)$, the analogous procedures derive

$$A^B(t+1) = \left(\frac{\mu L\Theta\,(1-\iota)\,(1+\Theta\,(1-\iota))}{(2L+1)\,\Theta\,(1-\iota)+1} + 1 \right) A^B(t). \tag{19}$$

[14] Considering (15), one may conjecture that if $N^A(t)$ and $N^B(t)$ were zero, $A^A(t+1) < A^B(t+1)$ might hold by taking a sufficiently large $M^B(t)$. Leapfrogging may then be able to occur simply through spillovers $M^B(t)$. However, this conjecture does not make sense. First, this example (of such a large $M^B(t)$) is not feasible in the space that we consider in the present model. Second, and more importantly, innovation in country A, $N^A(t-1) = M^B(t)$, not only contributes to the foreign knowledge, $A^B(t+1)$, through spillovers but also increases the domestic knowledge $A^A(t)$ in the previous period. This implies that $A^A(t+1)$ includes $N^A(t-1) = M^B(t)$ since it includes $A^A(t)$. As $N^A(t-1) = M^B(t)$ becomes large, by (7), $A^A(t+1)$ increases at a higher rate than, or at least the same rate as, $A^B(t+1)$. So, in this case of $N^A(t) = N^B(t) = 0$, for any large $M^B(t)$, $A^A(t+1) < A^B(t+1)$ is not possible. Thus, as long as we discern an identical equilibrium path, spillovers $M^B(t)$ by themselves cannot cause a reversal of $A^A(t) > A^B(t)$.

From (16) and (19), $A^A(t+1) > A^B(t+1)$ holds so long as $A^A(t) > A^B(t)$, given $\Theta(1-\iota) < 1$ and $\mu < 1$. This proves that $A^A(t) > A^B(t)$ cannot be reversed for the subsequent period when $\Theta(1-\iota) < 1$, whether either country is a leading country in the previous period $t-1$. ■

We now elaborate upon the intuition why the North–South economy cannot experience leapfrogging. There are potentially two sources of leapfrogging: knowledge growth from domestic innovation and spillovers from foreign innovation. In the North–South regime, however, the lagging South (country B) only has spillovers from $M^B(t)$ foreign innovations developed in the leading North (country A). No domestic innovations take place in the lagging country. Given that the leading North also gains from these innovations of $M^B(t)$ (which are included in $A^A(t)$) even more efficiently than, or at least equally to, the lagging South, the natural property works that *spillovers alone can only make the lagging South as innovative as, but not more innovative than, the leading North*. Thus, in the North–South regime, leapfrogging never takes place. As shown later, if the lagging country not only has spillovers from the leading country but also innovates by itself, leapfrogging will be possible.

Why then does the world fall into the North–South regime when $\Theta(1-\iota) < 1$? This condition intuitively requires that the expected discounted value for an innovation is fairly low. That is, it is too low for the lagging country B to innovate by itself.[15] In other words, where $\Theta(1-\iota)$ is higher, say, because of a lower imitation rate ι, the expected benefit from an innovation would be higher and thus innovation would be profitable, even for firms in the lagging country. Finally, we may summarize this by stating that: *when imitation in the lagging country is active, the world enters the North–South regime in which leapfrogging does not take place because the lagging country does not innovate alone.*

3.2 An illustration

To illustrate further the international dynamics of knowledge in the North–South regime, we assume that the leading country A has retained leadership in the past; i.e., $N^A(s) > 0 = N^B(s)$ and thus $A^A(s) > A^B(s)$ for $s = t,\ t-1,\ \cdots$. This consideration is reasonable given that Theorem 1 shows that leapfrogging never takes place in the North–South regime. The growth of knowledge follows (16) and (18) for any $s \geq t$. Define $\psi(t) = A^A(t)/(A^A(t) + A^B(t))$, which stands for the knowledge ratio for country A. We can derive the dynamic system for $\psi(t)$ as follows. Noting $A^A(t) > A^B(t)$,

$$\psi(t+1) = \frac{(a_1+1)\,\psi(t)}{1+(a_1+\mu a_2)\,\psi(t)} \text{ for } \psi(t) \in (0.5, 1), \tag{20}$$

where a_1 and a_2 are positive numbers determined by β, θ, ι, and L.[16] By applying the above procedures to the case of $\psi(t) \in (0, 0.5)$ where country B is the leading country,

[15] For ι, a higher imitation rate ι results in a smaller expectation for innovation firms to obtain profit. For β in Θ, a lower time preference β results in a higher interest rate $r(t)$, decreasing the discounted value for the profit. For θ^{-1} in Θ, a higher elasticity of substitution θ^{-1} implies a lower markup ratio $(1/1-\theta)$ and thus a lower profit $\pi(t)$.

[16] The formal definitions are:

$$a_1 \equiv \frac{2L\Theta(1-\iota)}{1+\Theta(1-\iota)} \text{ and } a_2 \equiv \frac{L\Theta(1-\iota)(1+\Theta(1-\iota))}{1+\Theta(1-\iota)(2L+1)}.$$

we can easily derive the following dynamic system:

$$\psi(t+1) = \frac{(1-\mu a_2)\,\psi(t) + \mu a_2}{1 + (1-\psi(t))\,(a_1 + \mu a_2)} \text{ for } \psi(t) \in (0, 0.5). \tag{21}$$

Note that $a_1 < 1$ and $a_2 < 1$ if $\Theta(1-\iota) < 1$. We thus can verify that so long as $\Theta(1-\iota) < 1$, the steady state is unique and higher than 0.5 for (20) and lower than 0.5 for (21).

Figure 1 illustrates the phase diagram for systems (20) and (21) with their steady states, ψ_A^* and ψ_B^*. As shown, any path starting in the situation where country A (B) is the leading country stably converges to a steady state; $\psi(s) > (<) 0.5$ for all $s > t$ if $\psi(t) > (<) 0.5$. Thus, this phase diagram visually shows that no leapfrogging occurs in the North–South regime.

3.3 North–North regime

Let us now consider the situation where both countries innovate (pattern (2)), which is realized when $\Theta(1-\iota) > 1$, as explained below. In this case, only the lagging country, B, manufactures. Then we have $N^A(t) > 0$ and $N^B(t) > 0$; $N(t+1) = N^A(t) + N^B(t)$. In contrast to the North–South regime, both countries behave like the North in the sense that the North generally innovates. Thus, we may refer to this specialization pattern as a North–North regime. Because innovation takes place in both countries, by the free-entry condition $V^A(t) = V^B(t) = 0$, the wages satisfy $w^A(t)/A^A(t) = w^B(t)/A^B(t)$. Manufacturing takes place only in country B, so that $\pi^A(t+1) < \pi^B(t+1)$ must hold, which implies $w^A(t) > w^B(t)$. Thus, the following inequality must hold in this case: $A^A(t) > A^B(t)$.

Taking into account the monopolistic price $p^B(t) = w^B/(1-\theta)$ in (5) and (6), the free-entry condition of $V^i(t) = 0$ with the Euler equation (3) implies

$$\beta\theta\,(1-\iota)\,\frac{E(t)}{N^A(t) + N^B(t)} = \frac{w^i(t)}{A^i(t)} \tag{22}$$

for each i. The interpretation of (22) is similar to that of (8) in the North–South regime. In the North–North regime, the labor market-clearing conditions are given by $L = N^A(t)/A^A(t)$ for the leading country A and $L = \left(N^B(t)/A^B(t)\right) + N(t)x^B(t)$ for the lagging country B, in which $x^B(t) = (1-\theta)\frac{E(t)}{N(t)w^B(t)}$ holds by using (4) and $p^B(t) = w^B/(1-\theta)$. With (22), these labor conditions determine the innovation flows as

$$N^A(t) = LA^A(t) \tag{23}$$

and

$$N^B(t) = \frac{1}{\Theta\,(1-\iota) + 1}\left(\Theta\,(1-\iota)\,LA^B(t) - LA^A(t)\right). \tag{24}$$

Equation (24) shows that the innovation flow in the lagging country $N^B(t)$ increases with the domestic knowledge stock $A^B(t)$ but decreases with the foreign knowledge stock $A^A(t)$. In order to ensure that the lagging country also innovates, $N^B(t) > 0$, by (24), we need to assume

$$\Theta\,(1-\iota) > \frac{A^A(t)}{A^B(t)} > 1, \tag{25}$$

which requires that the international technological gap, $A^A(t)/A^B(t)$, is not very large. To allow (25) to be feasible, we need to impose $\Theta(1-\iota) > 1$.

What if the knowledge gap, $A^A(t)/A^B(t)$, is larger than $\Theta(1-\iota)$? The specialization pattern then goes to case (3), where both countries are specialized such that the leading country innovates and the lagging country manufactures. We refer to this as a full North–South regime. In this case, the wage rate in the lagging country is determined by its labor market-clearing condition $L = N(t)x^B(t)$ as $w^B(t) = (1-\theta)E(t)/L$. The innovation flow in the leading country A $N^A(t)$ does not change from (23) while that in the lagging country B is zero ($N^B(t) = 0$). The condition for the full North–South regime is

$$\frac{A^A(t)}{A^B(t)} > \Theta(1-\iota) > 1. \tag{26}$$

To prove that leapfrogging may take place in the North–North regime, we leave the full North–South regime to Section 2.4 by imposing (25). In proving this, we suppose that country A retains leadership for two consecutive periods. That is, (25) holds for two periods, t and $t-1$. This implies that spillovers $M^B(t)$ are equal to $N^A(t-1) = LA^A(t-1)$ because innovations developed by country A in period $t-1$ all flow to the lagging country B.

By substituting (23) and (24) into (15) , with $M^A(t) = 0$ and $M^B(t) = LA^A(t-1)$, the growth of knowledge in the North–North regime follows

$$A^A(t+1) = \underbrace{LA^A(t)}_{N^A(t):\ \text{domestic innovation}} + A^A(t), \tag{27}$$

$$A^B(t+1) = \underbrace{\frac{\Theta(1-\iota)LA^B(t) - LA^A(t)}{\Theta(1-\iota)+1}}_{N^B(t):\ \text{domestic innovation}} + \underbrace{\frac{\mu L}{L+1}A^A(t)}_{M^B(t):\ \text{spillovers}} + A^B(t). \tag{28}$$

What is important here is that in the North–North regime, the lagging country B has two sources of knowledge growth, namely, domestic innovation $N^B(t)$ and spillovers from foreign innovation $M^B(t)$, which sharply contrast with the North–South regime where the lagging country does not innovate. By combining (27) and (28), we derive the international dynamics of knowledge as

$$\psi(t+1) = \frac{(L+1)\psi(t)}{\frac{\mu L}{L+1}\psi(t) + \left(\frac{\Theta(1-\iota)L}{\Theta(1-\iota)+1} + 1\right)}, \tag{29}$$

given $0.5 < \psi(s) < \frac{\Theta(1-\iota)}{1+\Theta(1-\iota)}$ for $s = t,\ t-1$ coming from (25).

Using (29), the following theorem formally proves the perpetual occurrence of leapfrogging as an equilibrium phenomenon.

Theorem 2 (Perpetual Leapfrogging in the North–North Regime) *Suppose* $\Theta(1-\iota) > A^A(s)/A^B(s) > 1$ *for* $s = t, t-1$. *Then, under dynamic optimization by the infinitely lived agent, the world is in the North–North regime in period t, where both the leading country and the lagging country innovate. In this case, neither country may be able to retain its technological leadership for infinite sequential periods; i.e., leapfrogging may take place repeatedly and perpetually along an equilibrium path. Specifically, this occurs if*

$$\mu > \frac{2(L+1)}{\Theta(1-\iota)+1}. \tag{30}$$

Proof. First, (25) ensures the North–North regime. The steady state of system (29) is uniquely given by

$$\psi^* = \frac{1}{\mu} \frac{L+1}{\Theta(1-\iota)+1},$$

which is less than 0.5 so long as (30) holds. If $\psi^* < 0.5$, given (29), $\psi(t)$ will stably decrease and eventually go below 0.5. This shows that when country A has leadership for two periods (t and $t-1$), it can never retain its leadership for infinite sequential periods. Put differently, technological leadership is always temporary. By symmetry, it is straightforward to show the opposite case where country B initially has leadership for two periods. This proves the occurrence of perpetual leapfrogging, taking into account the fact that for the case where a country initially has leadership for just one period, either the country retains leadership for two periods or is immediately leapfrogged. ■

Noting (30), we can see that Theorem 2 has an important implication concerning the likelihood of perpetual leapfrogging.

Proposition 1 *Leapfrogging may take place repeatedly and perpetually in the North–North regime if the efficiency of international spillovers μ is higher and/or if the imitation rate ι is lower.*

The key driving force behind perpetual leapfrogging is the dual growth engine of a lagging country. In the North–North regime, which results from a lower imitation rate, the lagging country both innovates and manufactures. Thus, the lagging country's knowledge accumulates not only through its own innovations but also through the flow of spillovers from the leading country's innovations. In this sense, the growth engine of knowledge in the lagging country is dual: innovating by itself and learning from abroad.[17] Conversely the leading country only innovates. Although it innovates faster than the lagging country, the knowledge growth in the leading country is driven only by domestic innovations. This creates the possibility of leapfrogging.

Needless to say, this is an extreme case as specialization takes place in the present model, which is a dynamic version of the Ricardian model. In reality, the leading country also manufactures foreign innovations (those in the lagging country) and may also learn from them. Therefore, we consider that this model captures only one particular aspect of real-world behavior. That is, lagging countries may have an advantage in international technology competition with the leading country because they can learn from the leader's active innovation as well as their own experience in innovation. However, this analysis does formally show that in a two-country model with dynamic optimization of the infinitely lived consumer, if a country can learn sufficiently from the foreign country in a North–North regime where imitation is less active in the lagging country, technological leadership perpetually alternates between the countries.[18]

[17] Recall that in the North–South regime, knowledge growth in the lagging country results only from foreign innovations, in which leapfrogging is not possible (Theorem 1).

[18] Given the historical fact that technology leadership has often shifted between countries, it is important to provide an extended case comprising more than two countries. We can then demonstrate that three or more countries on an equilibrium path can perpetually experience various forms of leapfrogging including, for example, growth miracles (Matsuyama 2007), in which the least productive country leapfrogs all rival countries with higher productivity levels in a single burst. Such growth miracles may take place sporadically or consecutively or in some complex combination. See Furukawa (2012) for a formal analysis.

3.4 An illustration

To obtain a graphic understanding, we again use a phase diagram. However, the configuration of the phase diagram depends on the history, i.e., which country was a leading country in the previous period. As this is simply a problem of visual complication, to clarify the illustration, we assume that innovation activities are completed within one period. Thus, the innovation value in (6) should be replaced by

$$V^i(t) = (1-\iota)\max\{\pi^A(t), \pi^B(t)\} - w^i(t)/A^i(t). \tag{31}$$

Define $\hat{\theta} \equiv \frac{\theta(1-\iota)}{1-\theta+\theta(1-\iota)}$. Noting that $\hat{\theta} > 0.5$ holds if (25) or (26) holds, we can describe the international dynamics of knowledge as follows.[19]

$$\psi(t+1) = \Phi(\psi(t)) \equiv \begin{cases} \frac{1}{L+1}\left(\mu L + \frac{\psi(t)}{1-\psi(t)}\right) & \text{for } \psi(t) \in (0, 1-\hat{\theta}) \\ \frac{(\hat{\theta}-(1-\mu))L+(1+(1-\mu)L)\psi(t)}{\hat{\theta}L+1+\mu L(1-\psi(t))} & \text{for } \psi(t) \in (1-\hat{\theta}, 0.5) \\ (L+1)\frac{\psi(t)}{1+\hat{\theta}L+\mu L\psi(t)} & \text{for } \psi(t) \in (0.5, \hat{\theta}) \\ (L+1)\frac{\psi(t)}{1+(1+\mu)L\psi(t)} & \text{for } \psi(t) \in (\hat{\theta}, 1) \end{cases}. \tag{32}$$

The equilibrium dynamic system Φ is autonomous and nonlinear. Figure 2 depicts the phase diagram of system Φ for $\mu < 2(1-\hat{\theta})$. There are two steady states, both of which are stable. For all initial points, technological leadership can never alternate internationally. In this case of $\mu < 2(1-\hat{\theta})$ where the spillovers are less efficient (small μ) and imitation is more active (high ι and thus small $\hat{\theta}$), the result is essentially identical to that in the North–South regime; that is, no leapfrogging takes place.

There are two subcases with (a) $\mu < \hat{\theta}^{-1}(1-\hat{\theta})$ and (b) $\hat{\theta}^{-1}(1-\hat{\theta}) < \mu$. In case (a), even if the advantage of the leading country is initially very small ($\psi(t)$ is around 0.5), the knowledge gap stably widens and the world economy finally converges to the steady state (ψ_i^{**}) as the full North–South regime without experiencing any leapfrogging. The two countries, even though both are North initially, will eventually split into North and South, in which the outcome is ultimately determined by the initial (slight) knowledge difference. In case (b), $\psi(t)$ converges to the steady state (ψ_i^*) in the North–North regime, in which case both countries can be North in the long run.

From this fact, we may state that in the world where leapfrogging never occurs, the initially lagging country eventually becomes the South, even though its initial disadvantage in knowledge stock is very small, if $\mu < \hat{\theta}^{-1}(1-\hat{\theta})$ holds; i.e., if the efficiency of international spillovers μ is lower and the imitation rate ι is higher. If μ is not very low and ι is not very high, even the lagging country can become innovative in the long run.

Most interestingly, Figure 3 depicts a typical path for the case in which $\mu > 2(1-\hat{\theta})$. Given that no steady state exists, the international knowledge fraction $\psi(t)$ will move perpetually back and forth between the two regimes $(0, 0.5)$ and $(0.5, 1)$. Finally, note that the condition of perpetual leapfrogging in the simplified model, $\mu > 2(1-\hat{\theta})$, is analogous to (30).

4 Concluding remarks

In this paper, we developed a two-region endogenous innovation model with the dynamic optimization of the infinitely lived consumer, in which knowledge diffuses internationally

[19] See an unpublished appendix to this paper available from the author upon request.

through imitation and foreign direct investment. The major finding is that technological leadership may shift internationally, perpetually moving back and forth between countries if imitation is less active and the spillovers are more efficient. Specifically, if imitation is active, in equilibrium, the world moves to a North–South regime where only the leading country innovates. In this regime, leapfrogging never arises. If imitation is less active, in equilibrium, a North–North regime arises where both countries innovate. In this regime, leapfrogging perpetually takes place along an equilibrium path if international spillovers are sufficiently efficient.

Our result is novel in at least two aspects. The first is to show that perpetual leapfrogging can occur along an equilibrium path, which is possible only in the North–North regime with efficient spillovers. The second is to perceive the endogenous selection of North–South and North–North models as a market equilibrium phenomenon. That is, the world works like a North–North (North–South) economy when imitation is less (more) active. The most striking implication is that active imitation in the lagging country may hamper its ability to leapfrog the leading country, even if imitation is an essential channel for knowledge spillovers. We then suggest that imitation is only useful for the lagging country to catch up with the leading country. Only when the lagging country shifts to an innovative economy in a North–North regime by discouraging imitation, leapfrogging can occur in equilibrium.

To grasp the essence of perpetual leapfrogging, we have left some important issues to discuss for future work. First, we have conceptualized essentially homogeneous countries. Departing from this, we would be able to investigate various patterns of leapfrogging, including one-time or terminal leapfrogging. Second, given heterogeneous countries, to consider which country finally prevails may attract policy-related researchers to leapfrogging issues. For example, the government of a country may affect the process of leapfrogging by means of policy, including subsidies, tariffs, competition policies, and institutional reforms. In pursuing this line of research, it could be interesting to investigate the Nash equilibrium in a policy game where each government maximizes domestic welfare. Third, as an alternative, it may be fruitful to relate the degree of international spillovers to the legal protection of intellectual property, a prominent issue in international relations. Strengthening the domestic level of intellectual property protection may or may not delay the timing of a county to leapfrog (or be leapfrogged). It may even deprive it of the opportunity to leapfrog. Finally, it would also be important to extend our discrete-time analysis to continuous time. In this study, leapfrogging is considered a nonlinear, discrete phenomenon, which would help understanding of the fundamental mechanism of leapfrogging. One may instead consider the leapfrogging mechanism in a continuous-time model. One possible way forward would be to focus on technological complementarity between countries. Spillovers from the leading country then combine with the backward technology of the lagging country, so leapfrogging should be more likely.

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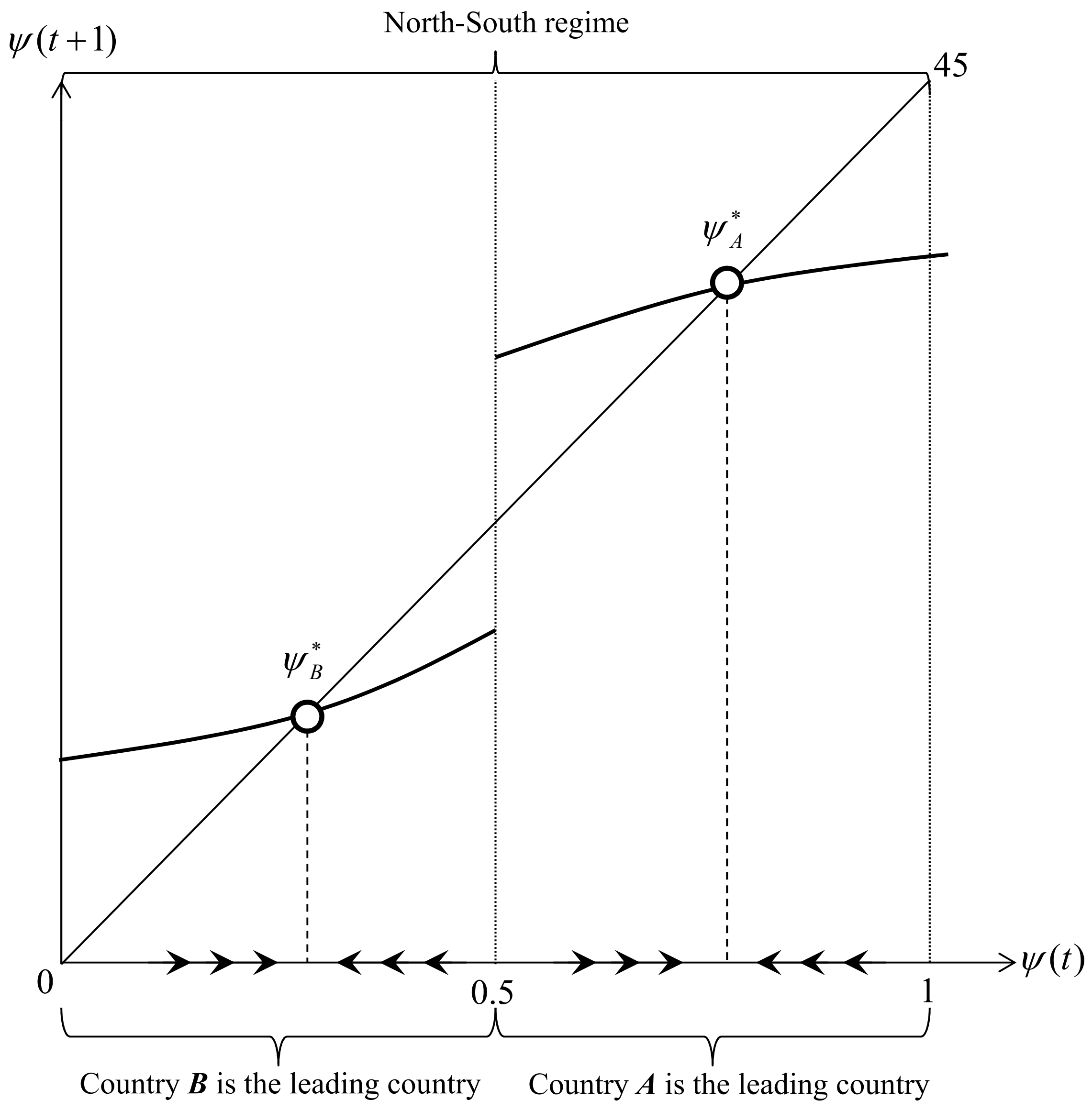

Figure 1: No leapfrogging in the North-South regime

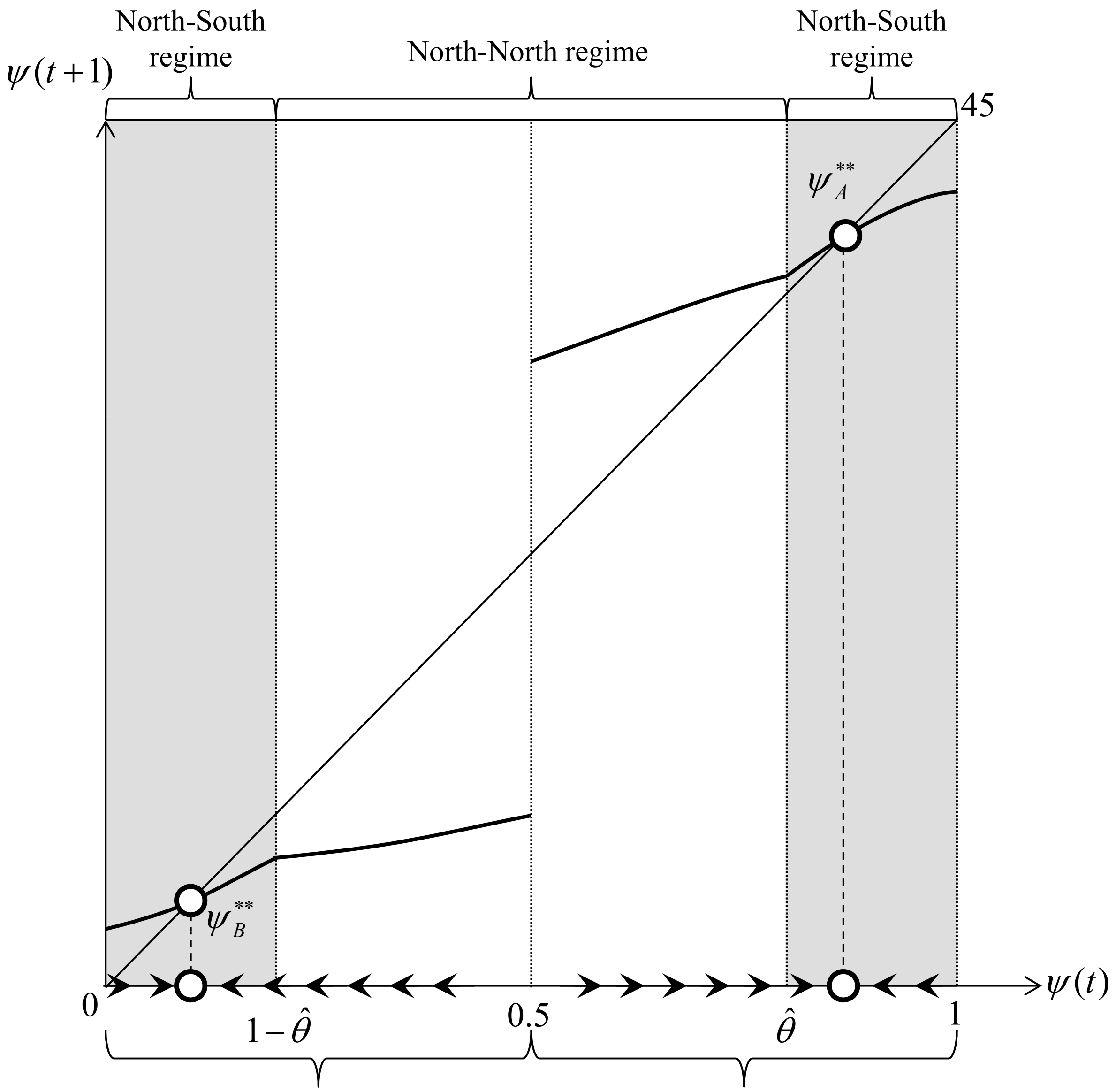

Figure 2: No leapfrogging in the North-South regime
(a) Converging to ψ_i^{**} as $\mu < (1-\hat{\theta})/\hat{\theta}$

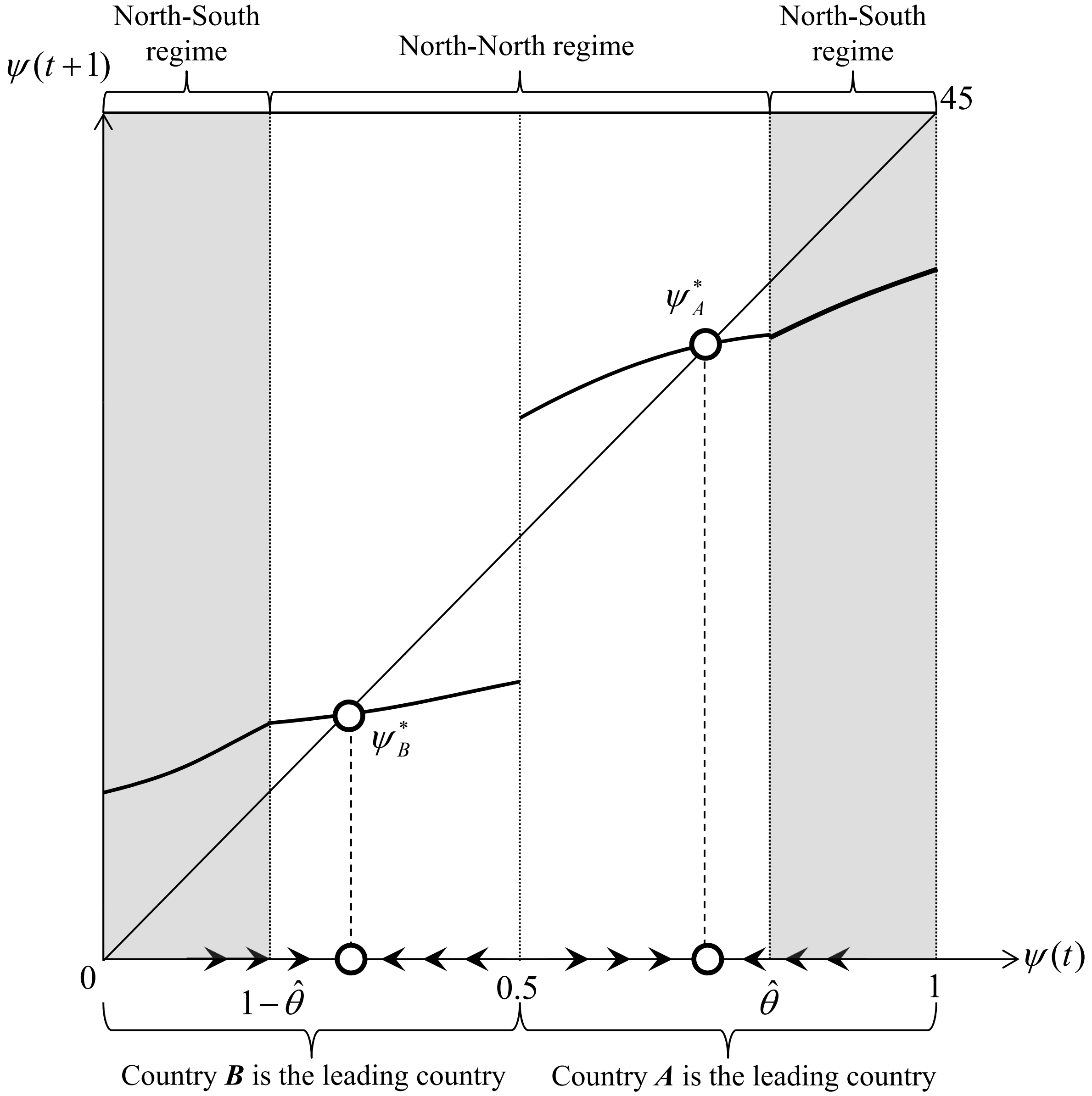

Figure 2: No leapfrogging in the North-North regime
(b) Converging to ψ_i^* as $\mu > (1-\hat{\theta})/\hat{\theta}$

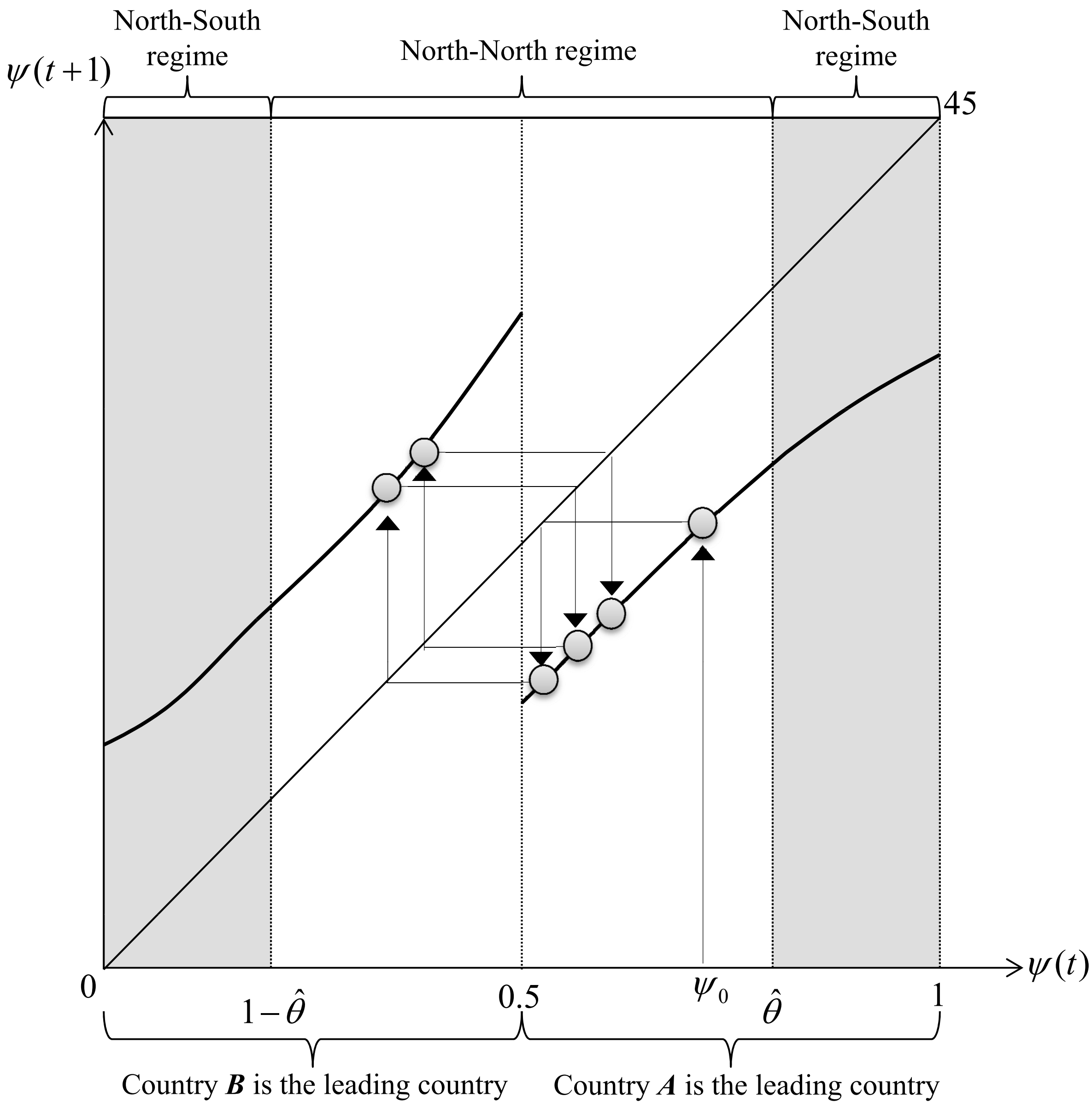

Figure 3: Perpetual leapfrogging in the North-North regime

Unpublished Appendix

In the North–North regime:

(A) Assume $\frac{\theta(1-\iota)}{1-\theta} > \frac{A^A(t)}{A^B(t)} > 1$. The free-entry condition (22) becomes

$$\theta\,(1-\iota)\,\frac{E(t)}{N^A(t)+N^B(t)} = \frac{w^i(t)}{A^i(t)}.$$

Given the labor condition for the leading country A, we have

$$N^A(t) = LA^A(t).$$

By the labor condition for the lagging country, $L = \left(N^B(t)/A^B(t)\right) + \left(N^A(t)+N^B(t)\right)x^B(t)$, with $x^B(t) = \frac{(1-\theta)E(t)}{(N^A(t)+N^B(t))w^B(t)}$, we thus have

$$N^B(t) = \frac{\frac{\theta(1-\iota)}{1-\theta}LA^B(t) - LA^A(t)}{1+\frac{\theta(1-\iota)}{1-\theta}}.$$

Noting $M^B(t) = N^A(t)$ here, by (15), we can have the dynamic system as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{\mu L\psi(t)+\hat{\theta}L+1} \text{ for } \psi(t) \in \left(0.5, \hat{\theta}\right),$$

where $\hat{\theta} \equiv \frac{\theta(1-\iota)}{1-\theta+\theta(1-\iota)}$. Note that $\hat{\theta} > 0.5$ holds in the North–North regime with $\frac{\theta}{1-\theta}\,(1-\iota) > 1$.

(B) Assume $\frac{1-\theta}{\theta(1-\iota)} < \frac{A^A(t)}{A^B(t)} < 1$. We can also derive

$$\psi(t+1) = \frac{(\hat{\theta}+(\mu-1))L+(1+(1-\mu)\,L)\,\psi(t)}{\hat{\theta}L+1+\mu L\,(1-\psi(t))} \text{ for } \psi(t) \in \left(1-\hat{\theta}, 0.5\right).$$

In the full North–South regime:

(A) Assume $\frac{A^A(t)}{A^B(t)} > \frac{\theta(1-\iota)}{1-\theta} > 1$. The leading country innovates following

$$N^A(t) = LA^A(t).$$

The lagging country receives spillovers $M^B(t) = N^A(t)$.[20] Then, the knowledge dynamics is as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{1+(1+\mu)\,L\psi(t)} \text{ for } \psi(t) \in \left(\hat{\theta}, 1\right).$$

(B) Assume $\frac{A^A(t)}{A^B(t)} < \frac{1-\theta}{\theta(1-\iota)} < 1$. We can easily have

$$\psi(t+1) = \frac{1}{L+1}\left(\mu L + \frac{\psi(t)}{1-\psi(t)}\right) \text{ for } \psi(t) \in \left(0, 1-\hat{\theta}\right).$$

[20] By using the labor market condition for the lagging country B and the free-entry condition, we can easily verify that

$$\frac{A^A(t)}{A^B(t)} > \frac{w^A(t)}{w^B(t)} = \frac{\theta\,(1-\iota)}{1-\theta} > 1$$

holds.